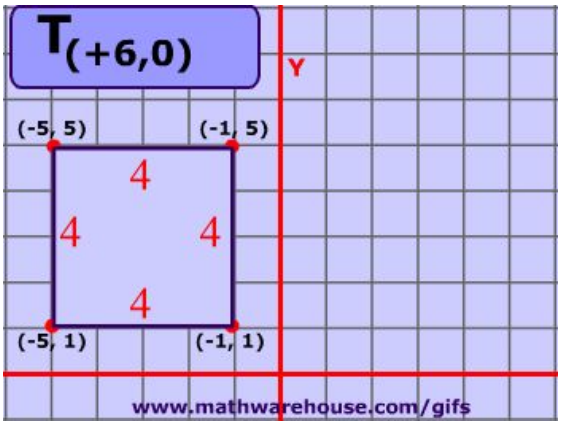
Geometric Transformations

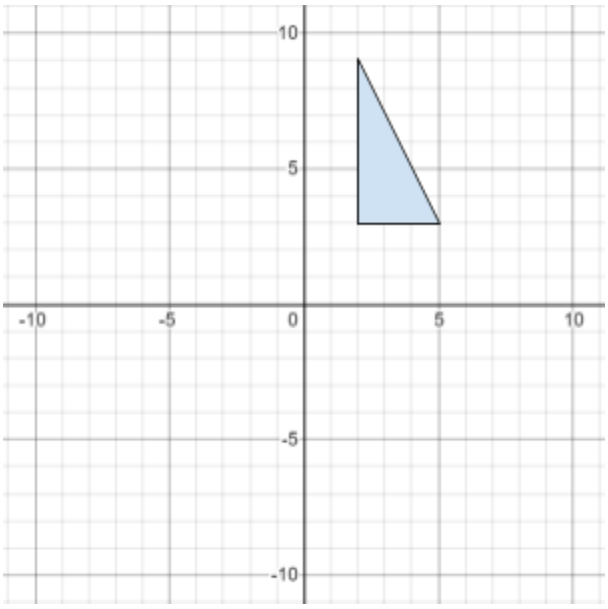
Transformations can help make your fashion designs pop. Below are four types of geometric transformations that could be useful in programming your LEDs. For example, you might want to make your picture animated by having it slide from one side of the frame to another. Perhaps you want your picture to spin like a merry-go-round or like a pinwheel. Maybe you want your picture to grow and/or shrink. You can use these geometric transformations in creating your fashion designs.

There are four transformations that are introduced below.

Translation: translations move an object without changing its orientation. They ***slide*** the object vertically, horizontally, or both. On the coordinate plane, this is done by adding (or subtracting) some number of units to the x-coordinate, the y-coordinate, or both the x- and y-coordinate of *every* point in an object.



For example, you can translate this rectangle 6 units to the right by adding 6 to the x-coordinate of every point in the triangle. This translation can be represented as (x, y) → (x + 6, y).

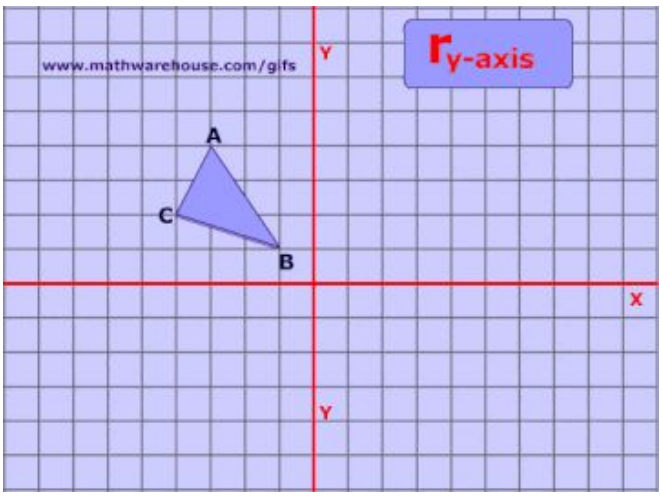


Now You Try! - Translations

1) Using the coordinate plane to the left, translate the triangle left 5 units.

2) Now, translate the original triangle down 7 units. Show how you would represent this transformation, then perform the transformation on the coordinate plane to the left.

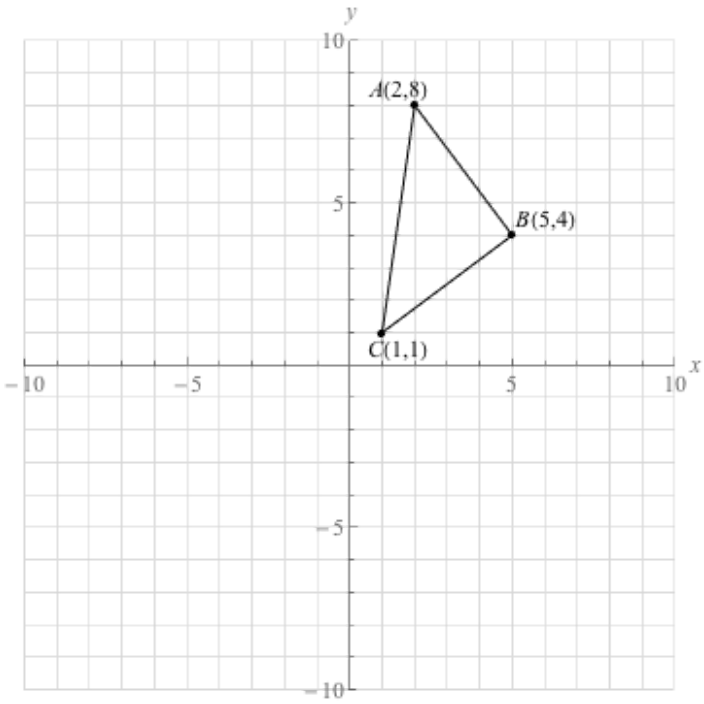
Reflection: a reflection creates a mirror image of an object. In a reflection, it is important to define where you want the mirror to be (your ***line of reflection***). Think about folding a piece of paper and creasing it. The crease acts as the line of reflection. If you had painted a triangle on one side of the paper, folded and creased it, you would have a reflected image of the original triangle on the other side of the crease. This is just like what happens on the coordinate plane. In the example below, the y-axis is the first fold, and the x-axis is the second fold.



For example, you can reflect this triangle across the y-axis by making the x-coordinate of every point in the object negative. In this case, the y-axis is your ***line of reflection***. The first transformation here can be represented as (x, y) → (-x, y). From the original to the third object, the reflection could be represented as (x, y) → (-x, -y), where you reflect across the y-axis *and then* across the x-axis.

Now You Try! - Reflections

1) Sketch the object reflected about the y-axis:

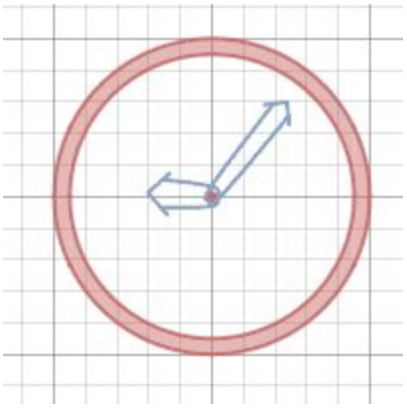


2) Now, reflect the original triangle about the x-axis. What is different about the end result when you reflect about the x-axis as compared to when you reflect about the y-axis?

Rotation: a rotation changes the orientation of an object by rotating it around a point. It is important to define your ***center of rotation,*** or the point around which you are rotating the object.

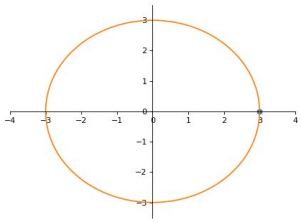
For example, you can rotate an object 90 degrees clockwise around the origin (***the center of rotation)*** by switching the x and y-coordinates of every point in the object and making the new y-coordinates all negative. This transformation can be represented as (x, y) → (y, -x)

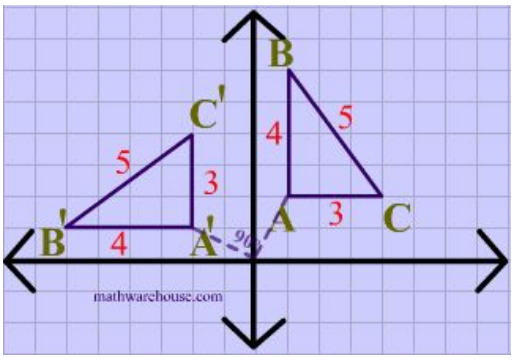
"Wait a minute! What does clockwise mean?"



Clockwise motion is motion that foellows the direction of a clock, like the image to the left shows.

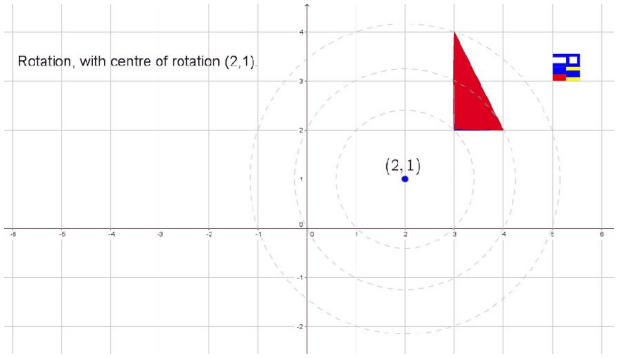
Similarly, counterclockwise motion is motion that goes the opposite direction of a clock, as is shown to the right.



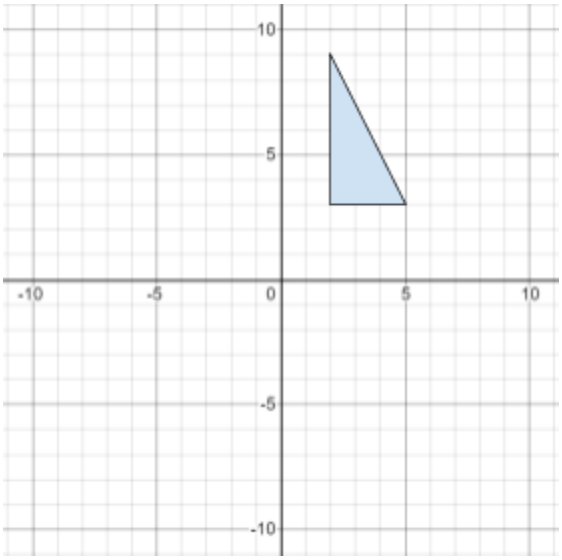


The diagram to the left is an example of counterclockwise rotation. Notice that the triangle is being rotated 90 degrees around the origin.

The diagram to the right shows what happens when we rotate around a point *other than the origin* or an angle *other than 90 degrees*.



Now You Try! - Rotations

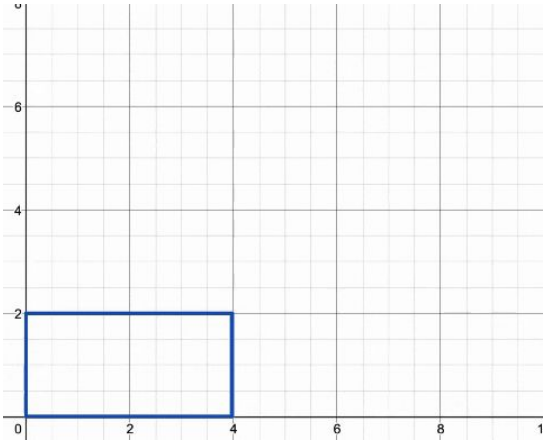


1) Sketch the object when it is rotated 90 degrees clockwise around the origin:

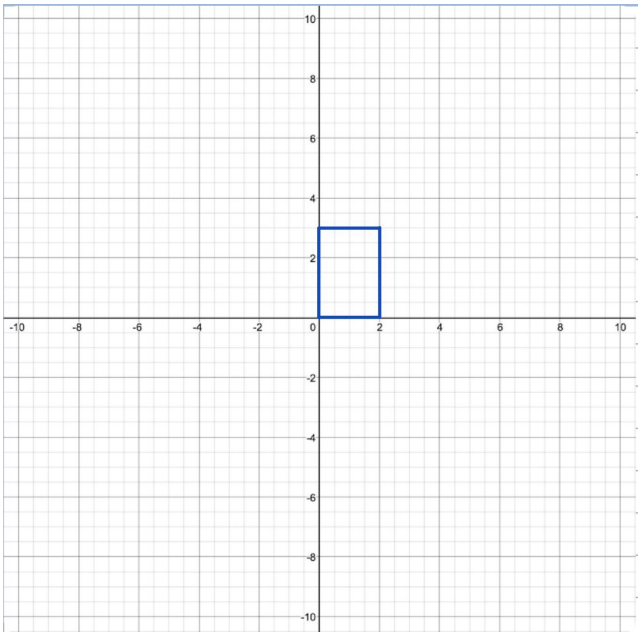
2) How would you rotate the object 90 degrees around the origin in the opposite direction? Show how you would represent this transformation, then perform the transformation on the coordinate plane above.

3) How would you rotate the triangle clockwise 180 degrees? Show how you would represent this transformation, then perform the transformation on the coordinate plane above.

Scaling: scaling an object means to make it bigger or smaller without distorting the picture. Think about a model car or a model of the Eiffel Tower. These are small replicas of larger objects; we call them scale models. Watch the image to the right as the original blue rectangle is scaled by a factor of 2 (each of the sides is doubled in length) to produce the larger red rectangle.



Now You Try! - Scaling

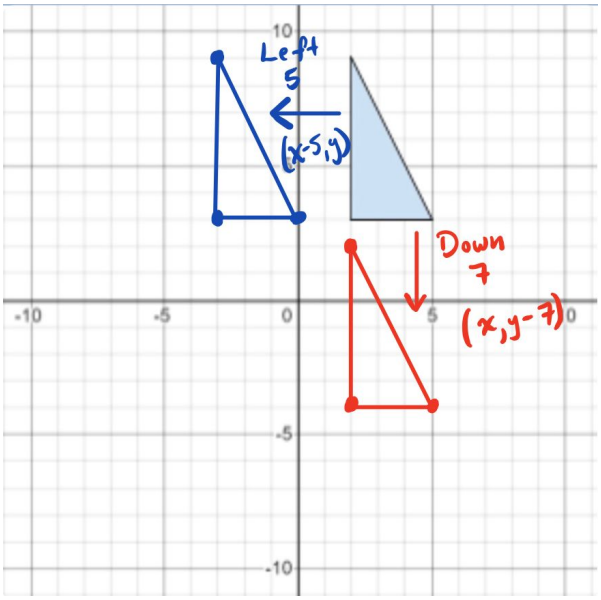


1) Sketch the object (blue) to the left when it is scaled up by 3 (this means triple the length of each side).

2) Sketch the original object (blue) when it is scaled down by 2 (this means divide the length of each side by 2).

Now You Try! Answer Key

Now You Try! - Translations



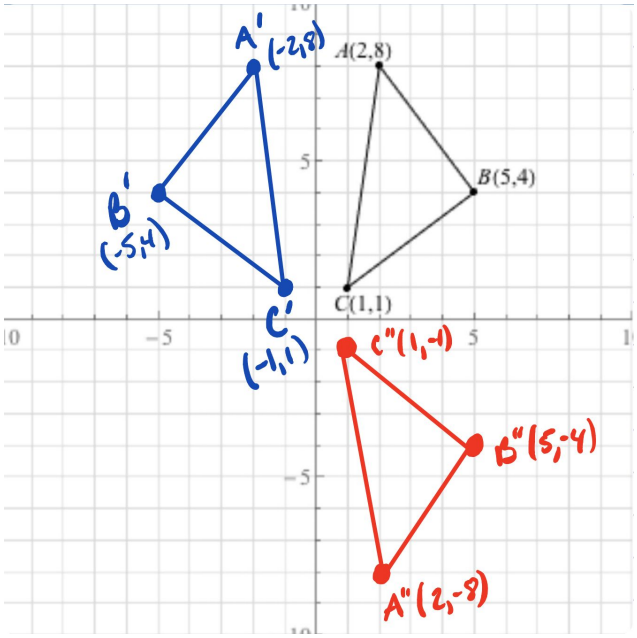
1) Using the coordinate plane to the left, translate the triangle left 5 units. Blue

2) Now, translate the original triangle down 7 units. Show how you would represent this transformation, then perform the transformation on the coordinate plane to the left. Red

Now You Try! - Reflections

1) Sketch the object reflected about the y-axis. Blue

2) Now, reflect the original triangle about the x-axis. What is different about the end result when you reflect about the x-axis as compared to when you reflect about the y-axis? Red

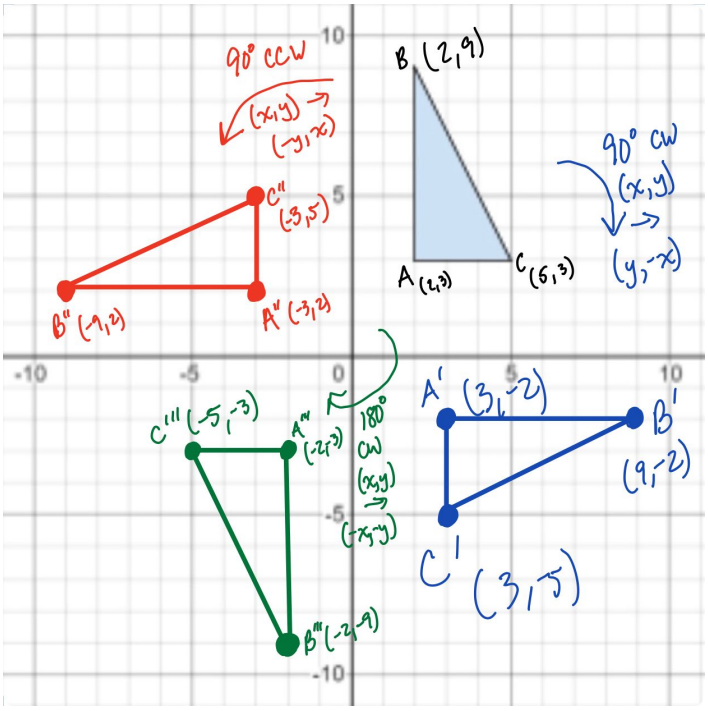


Now You Try! - Rotations

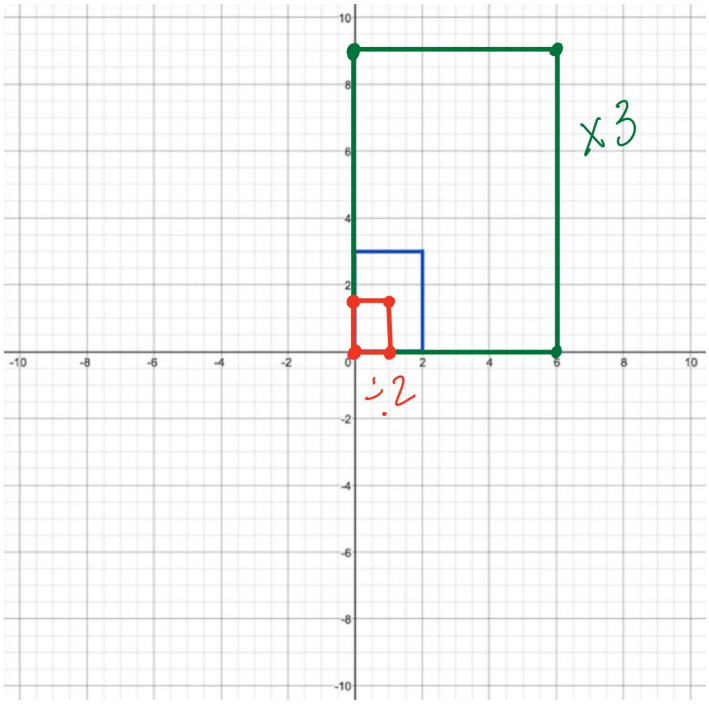
1) Sketch the object when it is rotated 90 degrees clockwise around the origin. Blue

2) How would you rotate the object 90 degrees around the origin in the opposite direction? Show how you would represent this transformation, then perform the transformation on the coordinate plane above. Red

3) How would you rotate the triangle clockwise 180 degrees? Show how you would represent this transformation, then perform the transformation on the coordinate plane above. Green



Now You Try! - Scaling



1) Sketch the object (blue) to the left when it is scaled up by 3 (this means triple the length of each side). Green

2) Sketch the original object (blue) when it is scaled down by 2 (this means divide the length of each side by 2). Red