DESIGN&PITCH CHALLENGE

Modeling Projectile Motion

Video game designers and programmers often rely on quadratic functions to make projectile motion appear realistic in their games. In this resource, you will explore how to use quadratic functions to model the projectile motion in your video game.

Knowing the following information can be helpful when modeling projectile motion:

- **The starting point for your projectile:** when and where does your projectile start moving and at what height?
- **The vertex for your projectile:** What is the maximum height of your projectile and when (time and distance) does it reach that height?
- **The end point for your projectile:** when and where does your projectile land and at what height?

Because you are the designer of your video game, you can research and select points that seem reasonable for your situation. Complete the following examples to see how two students modeled projectile motion differently for one video game.

Part 1: Sarah's Model of a Soccer Ball

Sarah is designing a video game involving soccer. She needs an equation that models the path of a kicked ball that has the following characteristics:

- The ball will reach a maximum height of approximately 5 yards.
- The ball will hit the ground after traveling approximately 36 yards (in the air) in 2.6 seconds.

Complete the following tables using these characteristics

Height of the Ball (yd) vs. Time (s)	
Time(s)	Height (yd)

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Height of the Ball (yd) vs. Distance Traveled (yd)	
Distance Traveled (yd)	Height (yd)

Sarah wants to use the factored form of a quadratic equation to model this situation.

Factored Form of a Quadratic Equation: f(x) = a(x-p)(x-q), where,

- p represents the x-coordinate of one x-intercept
- q represents the x-coordinate of the other x-intercept

Using the factored form of a quadratic equation, write an equation that represents the height of the ball vs. time. *Hint: you will need to substitute one of the points from the table (**not an x-intercept**) for x and f(x) to solve for a.*

Use Web Link - Desmos[1] to plot the points in the table on the coordinate plane and graph the equation. Is this equation a good fit for the path of the ball? Why or why not?

Using the factored form of a quadratic equation, write an equation that represents the height of the ball vs. the distance the ball travels. *Hint: you will need to substitute one of the points from the table (**not an x-intercept**) for x and f(x) to solve for a.*

Use Web Link - Desmos[1] to plot the points in the table on the coordinate plane and graph the equation. Is this equation a good fit for the path of the ball? Why or why not?

Part 2: Reviewing Sarah's Work

Rodrigo has been assigned to peer review Sarah's work. Sarah modeled the path of the ball in her game using a quadratic equation written in factored form (part 1). Rodrigo is more comfortable writing quadratic equations in vertex form. He decides to write and graph an equation in vertex form that models the path of Sarah's ball and then compare his graph to Sarah's.

Vertex Form of a Quadratic Equation: $f(x) = a(x-h)^2 + k$, where,

- h represents the x-coordinate of the vertex
- k represents the y-coordinate of the vertex

What equations, in vertex form, should Rodrigo graph for Sarah's game?

Height vs. Time

Using the vertex form of a quadratic equation, write an equation that Rodrigo could use to model the height of the ball vs. time. *Hint: to solve for a, you will need to substitute one of the points from the table (**not the vertex**) for x and f(x).*

Use Web Link - Desmos[1] to test the equivalence of Rodrigo's and Sarah's equations.

Height vs. Distance

Using the vertex form of a quadratic equation, write an equation that Rodrigo could use to model the height of the ball vs. the distance it travels. *Hint: to solve for a, you will need to substitute one of the points from the table (**not the vertex**) for x and f(x).*

Use Web Link - Desmos[1] to test the equivalence of Rodrigo's and Sarah's equations.

Reflection

How can this process help you model projectile motion in your video game? ***Note:** all examples in this resource use projectiles that start and stop on the x-axis and with a maximum height halfway between those points. If your projectile starts and stops different heights, be aware that your points may not lie on the same quadratic function.*

When would one form of a quadratic equation be more useful than another?

What could you do if one of your key features (e.g., x-intercept or vertex) doesn't fit your model?

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[1]Web Link - Desmos: https://www.desmos.com/calculator