Animated Contrasting Cases in Geometry (AC^{2}inG) Resource Guide

Thank you for your interest in our AC^{2}inG materials! This resource guide provides recommendations for implementation and use of these materials in your classroom. We hope that you and your students find the activities engaging and effective.

Explore the Activities: www.acinggeometry.org

What are the AC^{2}inG Materials? The AC^{2}inG Worked Example Pairs (WEPs) are supplemental, web-based instructional materials designed to utilize comparison to draw student attention to important mathematics underlying solution procedures related to **angles, transformation, the pythagorean theorem, and volume**. These materials are best used in 8th grade mathematics classrooms or with students learning material aligned with the 8th grade geometry Common Core State Standards for Mathematics (CCSSM). The AC^{2}inG materials are not designed for use as primary instructional materials. On the following pages an activity guide for each unit breaks down content included in each WEP.

Why comparison? Research suggests that the use of comparison in mathematics classrooms can support students in developing their procedural fluency and flexibility (Rittle-Johnson & Star, 2009). Additionally, opportunities to analyze and critique the reasoning of others, one of the standards for mathematical practice included in the Common Core State Standards for Mathematics, is an important skill set for students to develop. For students who may be hesitant to critique the arguments of their peers, utilizing the arguments of fictitious characters alleviates that hesitancy.

Description of AC^{2}inG Worked Example Pair Types		
WEP Type	**WEP Description**	**WEP Goal**
Why does it work?	One problem solved using two different, correct methods	Develop conceptual understanding or generalize underlying mathematics
How do they differ?	Two different problems solved using two different, correct methods	Highlight the relationship between solution methods and their common underlying mathematics
Which is better?	One problem solved using two different, correct methods	Promote student agency in selecting personally meaningful solution strategies
Which is correct?	One problem solved using one correct and one incorrect method	Highlight, and warn against, common errors and misconceptions

How can I use the AC^{2}inG Materials?

Prior to implementation of each WEP, provide instruction on relevant prerequisite knowledge. Then, in small groups or pairs, allow approximately 15 - 20 minutes for students to work through each WEP and complete the associated discussion questions. Students will watch a self-paced animation of two characters working a geometry problem. They will then view the two approaches side-by-side and work through them line-by-line, directly comparing the two approaches to make sense of the mathematics. After finishing the WEP, allow students time to work on the discussion questions. Printable PDF copies of discussion question pages are available on our website. Follow each WEP activity with a class discussion guided by the discussion questions to solidify the vocabulary, mathematical concepts, and key takeaways of the WEP. Each WEP has an associated WEP Type described in more detail below. Use the different descriptions and goals of the WEP types to guide the introduction of the activity as well as the follow up discussion. We suggest using no more than two WEPs per day and interspersing WEP activities with additional instruction and practice.

Key Activity Design Features

1. WEPs **displayed side-by-side** for ease of comparison
2. **Diverse characters** that present solution strategies
3. **Formal mathematical notation** displayed next to speech bubbles written in **relatable language** to facilitate student understanding
4. **Animations** display steps one at a time, draw student attention to underlying mathematics, and highlight relevant diagram components
5. Consistent **color coding** emphasizes connections between diagrams, mathematical notation, and verbal descriptions of solutions
6. **Discussion questions** encourage students to compare and contrast the worked solution methods, practice their understanding, and make generalizations
7. **Summary pages** highlight key mathematical takeaways of each WEP

Rittle-Johnson, B. & Star, J. R. (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology, 101*(3), 529-544.

AC^{2}inG Angles Unit Activity Guide

Angles Unit Content Standard(s):

CCSSM.8.G.A.5 - Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

AC^{2}inG Learning Goals:

1. Define and understand supplementary, vertical, corresponding, alternate interior, and same side interior angles
2. Apply understanding of angle relationships to find a missing angle measures when parallel lines are cut by a transversal
3. Define and understand the Triangle Sum and Exterior Angle Theorems

Angles Unit Content Breakdown by WEP

WEP	Content	Type	Key Takeaways	Prerequisite Knowledge	Relevant Vocabulary
A.1	Supplementary & Vertical Angles	Why does it work?	Vertical angles are congruent	Angle label and measure notation Angle congruence vs. measure equality Supplementary angles add to 180º	Supplementary Angles Vertical Angles
A.2	Corresponding Angles	How do they differ?	Corresponding angles are congruent	Supplementary angles add to 180º (A.1) Definition of parallel lines Definition of transversals	Parallel Lines Transversal Supplementary Angles Corresponding Angles
A.3	Alternate Interior & Same Side Interior Angles	How do they differ?	Same side interior angles are supplementary Alternate interior angles are congruent	Supplementary angles add to 180º (A.1) Definition of vertical angles (A.1) Vertical angles are congruent (A.1) Definition of corresponding angles (A.2) Corresponding angles are congruent (A.2)	Parallel Lines Transversal Supplementary Angles Vertical Angles Corresponding Angles
A.4	Missing Angles	Which is better?	Some methods are more efficient than others when solving for missing angles	Supplementary angles add to 180º (A.1) Vertical angles are congruent (A.1) Definition of and relationships between all angles formed by intersecting lines	Parallel Lines Transversal Supplementary Angles Vertical Angles Alternate Interior Angles
A.5	Triangle Angle Sum Theorem	Why does it work?	The angles interior to a triangle add to 180º	Angles that form a line add to 180º Alternate interior angles are congruent (A.3)	Parallel Lines Transversal Alternate Interior Angles
A.6	Exterior Angles	Which is correct?	The measure of an angle exterior to a triangle is equal to the sum of the two opposite interior angles	Definition of exterior angles Supplementary angles add to 180º	Supplementary Angles Exterior Angles Interior Angles

AC²inG Transformations Unit Activity Guide

Transformations Unit Content Standard(s):

CCSSM.8.G.A.2 - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

CCSSM.8.G.A.3 - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CCSSM.8.G.A.4 - Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

AC²inG Learning Goals:

1. Create algebraic and visual understanding of translations, reflections, rotations, and dilations
2. Defining and understanding similar and congruent figures using transformations

Transformations Unit Content Breakdown by WEP

WEP	Content	Type	Key Takeaways	Prerequisite Knowledge	Relevant Vocabulary
T.1	Translation	Why does it work?	Horizontal translations are equivalent to adding/subtracting from the x-coordinate Vertical translations are equivalent to adding/subtracting from the y-coordinate	Plot points on the Cartesian plane	Translation x-coordinate y-coordinate
T.2	Reflection	How do they differ?	Reflections over the x-axis negate the y-coordinate Reflections over the y-axis negate the x-coordinate	Plot points on the Cartesian plane	Reflection Axis
T.3	Rotation	Which is better?	Clockwise 90º rotations are equivalent to transforming (x,y) to (y,-x)	Plot points on the Cartesian plane Clockwise vs. counterclockwise 90º rotations	Rotations Clockwise Counterclockwise
T.4	Dilation	Which is correct?	Dilations about the origin are equivalent to multiplying both coordinates by the factor Dilations preserve angle measures	Plot points on the Cartesian plane Measuring angles	Dilations
T.5	Verify Congruence	Why does it work?	Reflections, rotations, and translations preserve congruence	Definition of triangle congruence Reflections in the Cartesian plane (T.2) Rotations in the Cartesian plane (T.3)	Triangle Congruence Reflections Rotations
T.6	Verify Similarity	Why does it work?	Series of reflections, rotations, translations, and dilations produce similar figures	Definition of triangle similarity Rotations in the Cartesian plane (T.3) Dilations about the origin (T.4)	Triangle Similarity Rotations Dilations

AC^{2}inG Pythagorean Theorem Unit Activity Guide

Pythagorean Theorem Unit Content Standard(s):
CCSSM.8.G.B.6 - Explain a proof of the Pythagorean Theorem and its converse.
CCSSM.8.G.B.7 - Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
CCSSM.8.G.B.8 - Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

AC^{2}inG Learning Goals:

1. Understanding when the Pythagorean Theorem works
2. Applying the Pythagorean Theorem to find the missing sides of right triangles
3. Understanding Pythagorean Triples
4. Drawing connections between the distance formula and the Pythagorean Theorem

Pythagorean Theorem Unit Content Breakdown by WEP					
WEP	**Content**	**Type**	**Key Takeaways**	**Prerequisite Knowledge**	**Relevant Vocabulary**
PT.1	Right and Obtuse Triangles	Why does it work?	The Pythagorean Theorem works only with right triangles	Pythagorean Theorem Definition of right triangles Definition of obtuse triangles	Right Triangle Obtuse Triangle
PT.2	Missing Side Length	Which is correct?	The Pythagorean Theorem can be used to find the missing side lengths of right triangles	Pythagorean Theorem	Right Triangle
PT.3	Connections to Distance Formula	Why does it work?	The Pythagorean Theorem can be used to find the distance between two points Establish connections between the distance formula and Pythagorean Theorem	Pythagorean Theorem Distance formula Plot points on the Cartesian plane	Right Triangle Hypotenuse
PT.4	Ladder Application	Which is correct?	The Pythagorean Theorem can be used to solve real-world problems involving right triangles	Pythagorean Theorem	Right Triangle Hypotenuse
PT.5	Plane Application	Why does it work?	The Pythagorean Theorem and distance formulas can be used to solve real-world problems	Pythagorean Theorem Distance Formula (PT.3)	
PT.6	Pythagorean Triples	How do they differ?	Pythagorean triples are sets of three numbers that satisfy the Pythagorean Theorem	Pythagorean Theorem	Pythagorean Triple

AC^{2}inG Volume Unit Activity Guide

Volume Unit Content Standard(s):
CCSSM.8.G.C.6 - Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

AC^{2}inG Learning Goals:
1. Comparing volume and surface area
2. Analyzing cylinders: volume, effects of scaling the height & radius
3. Finding the volume of composite figures

Volume Unit Content Breakdown by WEP					
WEP	**Content**	**Type**	**Key Takeaways**	**Prerequisite Knowledge**	**Relevant Vocabulary**
V.1	Surface Area vs. Volume	Which is correct?	Emphasize the differences between volume and surface area	Surface area of a rectangular prism Volume of a rectangular prism	Volume Surface Area Rectangular Prism
V.2	Volume of a Cylinder	Why does it work?	Connect the formula for the volume of a cylinder to the area of its base and its height	Volume of a cylinder Area of a circle	Cylindrical Base
V.3	Scale Cylinder Height	How do they differ?	The height of a cylinder is directly proportional to its volume	Volume of a cylinder (V.2) Height of a cylinder (V.2)	Cylinder Radius Volume Cone
V.4	Scale Cylinder Radius	How do they differ?	The radius of a cylinder is quadratically proportional to its volume	Volume of a cylinder (V.2) Radius of a cylinder (V.2)	Cylinder Radius Volume
V.5	Composite Figures	Which is better?	Demonstrate multiple solution pathways to find the volume of a single composite figure	Volume of a rectangular prism (V.1) Volume of a triangular prism Volume of a trapezoidal prism	Composite Figure Trapezoidal Prism Rectangular Prism Triangular Prism Volume